Balanced imputation for Swiss cheese nonresponse

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The Use of R in Official Statistics

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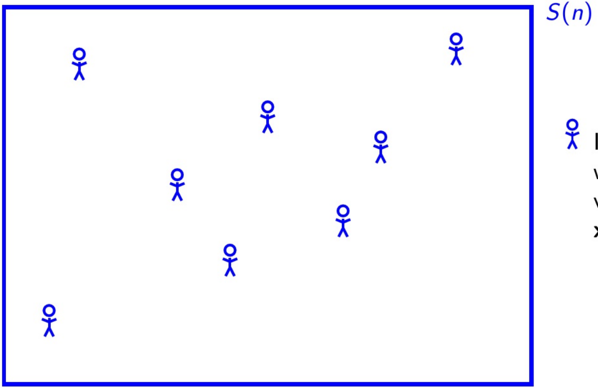
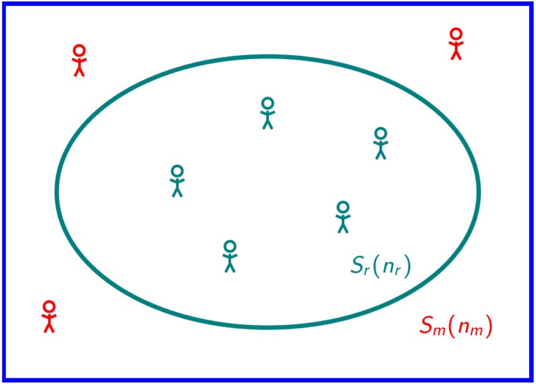
Overview

The nonresponse



Individual for whom we want to observe J variables to obtain vector $\mathbf{x}_k = (x_{1k}, x_{2k}, \dots, x_{Jk})^\top$.

The nonresponse



$S(n)$

- Respondent: $\mathbf{x}_k = (x_{1k}, x_{2k}, \ldots, x_{Jk})^\top$ is fully observed.
- Non-respondent: $\mathbf{x}_k = (x_{1k}, x_{2k}, \ldots, x_{Jk})^\top$ contains at least one missing or not usable value.

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Figure 2: The univariate case: a slice of Swiss cheese with holes.

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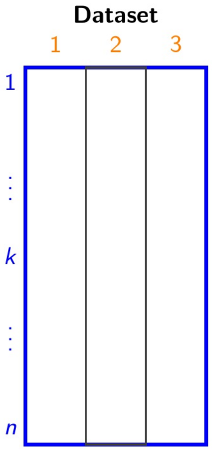
Figure 2: The univariate case: a slice of Swiss cheese with holes.

Figure 3: The multivariate case: a Swiss cheese with holes.

Notations of Swiss cheese nonresponse

- Population U of size N.
- J variables of interest.
- Sample $S \subset U$ of size n.
- For each unit $k \in S$: $\mathbf{x}_k = (x_{k1}, \dots, x_{kj}, \dots, x_{kJ})^\top$.



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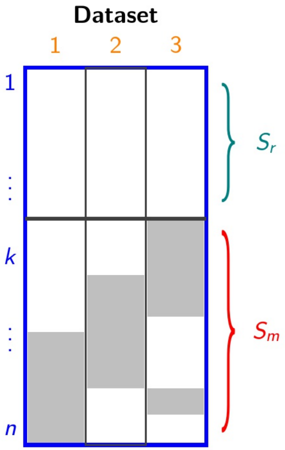
- For each unit $k \in S$: $\mathbf{x}_k = (x_{k1}, \dots, x_{kj}, \dots, x_{kJ})^\top$.

- Sample $S_r \subset S$ of size n_r contains completely observed units.
- Sample $S_m \subset S$ of size n_m contains units with at least one missing or not usable value.
- $S_r \cup S_m = S$ and $n_r + n_m = n$.
- Not monotone nonresponse.



Requirements of the imputation method

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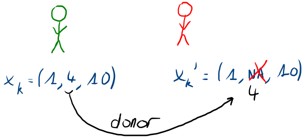

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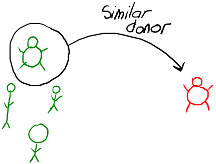

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(iii) If the observed values of the nonrespondents were imputed, the total estimator of each variable should remain unchanged.

Imputation probabilities

- Let $\boldsymbol{\psi} = (\psi_{uv})$ denote the matrix of size $n_r \times n_m$ containing imputation probabilities, where $(u, v) \in S_r \times S_m$.
- ψ_{uv}: probability that the respondent $u \in S_r$ gives its values to the nonrespondent $v \in S_m$.

Nonrespondents

Respondents

$$\boldsymbol{\psi} = \begin{pmatrix} \psi_{11} & \psi_{12} & \psi_{13} \\ \psi_{21} & \psi_{22} & \psi_{23} \\ \psi_{31} & \psi_{32} & \psi_{33} \\ \psi_{41} & \psi_{42} & \psi_{43} \end{pmatrix}$$

1 1 1

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3. If the observed values of the nonrespondents were imputed, the total estimator of each variable should remain unchanged:

$$\sum_{v \in S_m} r_{vj} \underbrace{\sum_{u \in S_r} \psi_{uv} x_{uj}}_{\text{imputed value of } x_{vj}} = \sum_{v \in S_m} r_{vj} x_{vj},$$

where r_{vj} is 1 if unit v responded to variable j and 0 otherwise.

Imputation probabilities

Steps to obtain final matrix ψ:

Step 1. Initialization of ψ:

$$\psi_{uv} = \begin{cases} \frac{1}{K} & \text{if } u \in \text{knn}(v), \\ 0 & \text{otherwise.} \end{cases}$$

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Step 2. Update $\boldsymbol{\psi}$ using an algorithm of calibration proposed by Deville and Särndal (1992) in order to satisfy requirements 1-3.

Imputation matrix

Matrix of imputation probabilities:

$$\psi = \begin{pmatrix} 0 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0 \\ 0.3 & 0 & 0.4 \\ 0.2 & 0 & 0.1 \end{pmatrix} \quad \rightarrow$$

Imputation matrix:

$$\phi = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Element $\phi_{uv} = 1$ means that missing values of nonrespondent v will be imputed by values of respondent u:

$$x^*_{uj} = \sum_{v \in S_r} \phi_{uv} x_{vj}.$$

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Step to obtain final matrix ϕ:

Step 3. Compute ϕ using a stratified sampling method such that:

- Columns of ψ correspond to stratum.
- Balancing constraints of requirement 3. is satisfied.

Example with the R software

```
# Download the package SwissCheese
# library(devtools)
# install_github("EstherEustache/SwissCheese@master")
# library(SwissCheese)

# Dataframe with NA values
Sm <- as.vector(attr(stats::na.omit(X_NA),"na.action"))
Sm

## [1] 18 21 29  5  1 10 17 20

Sr <- which(!(1:nrow(X) %in% Sm))
Sr

##  [1]  2  3  4  6  7  8  9 11 12 13 14 15 16 19 22 23 24 25 26 27 28
```

```r
# Nonrespondents
head(X_NA[Sm,])
##            V1       V2 V3
## [1,]       NA 49.48603  1
## [2,]       NA 36.05060  0
## [3,]       NA 19.34894  0
## [4,] 21.12337       NA  0
## [5,] 36.64376 47.15358 NA
## [6,] 23.75826 33.93555 NA
```

```r
# Respondents
head(X_NA[Sr,])
##            V1       V2 V3
## [1,] 47.19283 57.77238  1
## [2,] 42.91603 56.86644  1
## [3,] 57.71289 77.52506  1
## [4,] 40.32247 53.35982  1
## [5,] 52.91569 64.01816  1
## [6,] 63.35500 67.27140  1
```

```
## Swiss cheese imputation ##
SW <- swissCheeseImput(X = X_NA, d = NULL, k = NULL,
                       tol = 1e-3, max_iter = 50)
```

```
###---Optimal number of neighbors considered
SW$k

## [1] 4

###---The nonrespondent imputed
head(SW$X_new[Sm,])

##            V1       V2 V3
## [1,] 36.93283 49.48603  1
## [2,] 29.43678 36.05060  0
## [3,] 17.39018 19.34894  0
## [4,] 21.12337 45.00952  0
## [5,] 36.64376 47.15358  0
## [6,] 23.75826 33.93555  0
```

Bibliography

Deville, J.-C. and Särndal, C.-E. (1992). Calibration estimators in survey sampling. *Journal of the American Statistical Association*, 87:376–382.

Hasler, C. and Tillé, Y. (2016). Balanced k-nearest neighbor imputation. *Statistics*, 105:11–23.