

A Sparse Matrix Tool in R for Multidimensional Hierarchical Aggregation

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Abstract: Commonly, tabular aggregations involve hierarchical classifications (hierarchies) that follow a tree structure. Free hierarchies are more advanced. Then, each code can be specified as a formula involving other codes. Addition or union is specified by the plus sign and subtraction or compliment is specified by the minus sign. Statistics Norway's modernized calculations of municipal accounts involve complicated free hierarchies. The intention was to do the computations by an implementation of the Validation and Transformation Language (VTL), but this approach turned out to be too inefficient. Instead, a general hierarchical computing method was implemented in R as the function `HierarchyCompute` in the package `SSBtools`. Hierarchies and crossed hierarchies are represented by sparse matrices according to the `Matrix` package. The aggregates are obtained by matrix multiplication. The matrix representation therefore makes it easy to see how each output code is calculated from the input codes. A spin-off from the general implementation is that hierarchy specifications, usual in statistical disclosure control software, can be managed. This has been utilized in the package `SmallCountRounding`, which can be used to protect frequency tables.

- R package `SSBtools`
- Function `HierarchyCompute()`



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[See main figures for these statistics](#) | [Select other tables](#)

Municipal accounts

12367: Detailed accounting figures in the operational accounts, by accounting scope, function and type (M) 2015 - 2019



Choose table

Vindusklipp



Choose variables



Show result

List view

About table

Contents

Mandatory *

Select all

Deselect all

Total 1 Selected 1

Sum (NOK 1000)

Year

Mandatory *

Select all

Deselect all

Total 5 Selected 1

2019

2018

2017

2016

2015

Region

Mandatory *

Select all

Deselect all

Search

Beginning of word

Total 519 Selected 0

EAK The whole country

EAKUO The whole country except Oslo

0101 Halden

0104 Moss

0105 Sarpsborg

0106 Fredrikstad

Contents

Mandatory *

Select all

Deselect all

Total 1 Selected 1

Sum (NOK 1000)

Year

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Total 5 Selected 1

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EAKUO The whole country except Oslo

0101 Halden

0104 Moss

0105 Sarpsborg

0106 Fredrikstad

0111 Hvaler

0118 Aremark

Mandatory *

Accounting scope

Select all

Deselect all

Total 2 Selected 2

Municipality/county authorities

Municipality excluded municipal units, inter-mun

Function

Mandatory *

Select all

Deselect all

Primary and secondary schools

Select values from group

Total 6 Selected 1

202 Primary and lower secondary schools

213 Adult education at primary and lower second

215 Out of school care

222 School buildings

223 School transport

FGK8b Primary and secondary schools, school bu

Type

Mandatory *

Select all

Deselect all

Search

Beginning of word

Total 35 Selected 1

Adjusted gross operating expenditure

Gross operating expenditure

Net operating expenditure

Wages and salaries exclusive sick pay reimburs

Sick pay reimbursement

Pension premiums and taxable insurance sche

Employers' national insurance contributions

Show table

Number of selected data cells are: 10
(maximum number allowed is 300 000)

To the Top

12367: Detailed accounting figures in the operational accounts , by region, accounting scope , function, type, contents and year

Show table About table

Edit and Calculate Save table as Table - Layout 2

csv xlsx

+ Hide empty rows

+ Save your query

				Sum (NOK 1000)
				2019
EAK The whole country	Municipality/county authorities	FGK8b Primary and secondary schools, school buildings, school transport and adult educaton	Sick pay reimbursement	3 269 578
	Municipality excluded municipal units, inter-municipal company and cooperation	FGK8b Primary and secondary schools, school buildings, school transport and adult educaton	Sick pay reimbursement	3 251 842
EAKUO The whole country except Oslo	Municipality/county authorities	FGK8b Primary and secondary schools, school buildings, school transport and adult educaton	Sick pay reimbursement	2 927 529
	Municipality excluded municipal units, inter-municipal company and cooperation	FGK8b Primary and secondary schools, school buildings, school transport and adult educaton	Sick pay reimbursement	2 909 793
0101 Halden	Municipality/county authorities	FGK8b Primary and secondary schools, school buildings, school transport and adult educaton	Sick pay reimbursement	19 042
	Municipality excluded municipal units, inter-municipal company and cooperation	FGK8b Primary and secondary schools, school buildings, school transport and adult educaton	Sick pay reimbursement	19 042
0104 Moss	Municipality/county authorities	FGK8b Primary and secondary schools, school buildings, school transport and adult educaton	Sick pay reimbursement	13 093
	Municipality excluded municipal units, inter-municipal company and cooperation	FGK8b Primary and secondary schools, school buildings, school transport and adult educaton	Sick pay reimbursement	12 557
0105 Sarpsborg	Municipality/county authorities	FGK8b Primary and secondary schools, school buildings, school transport and adult educaton	Sick pay reimbursement	33 010
	Municipality excluded municipal units, inter-municipal company and cooperation	FGK8b Primary and secondary schools, school buildings, school transport and adult educaton	Sick pay reimbursement	33 010

Footnotes

The table shows detailed accounting figures for revenues and expenditures in the municipalities.

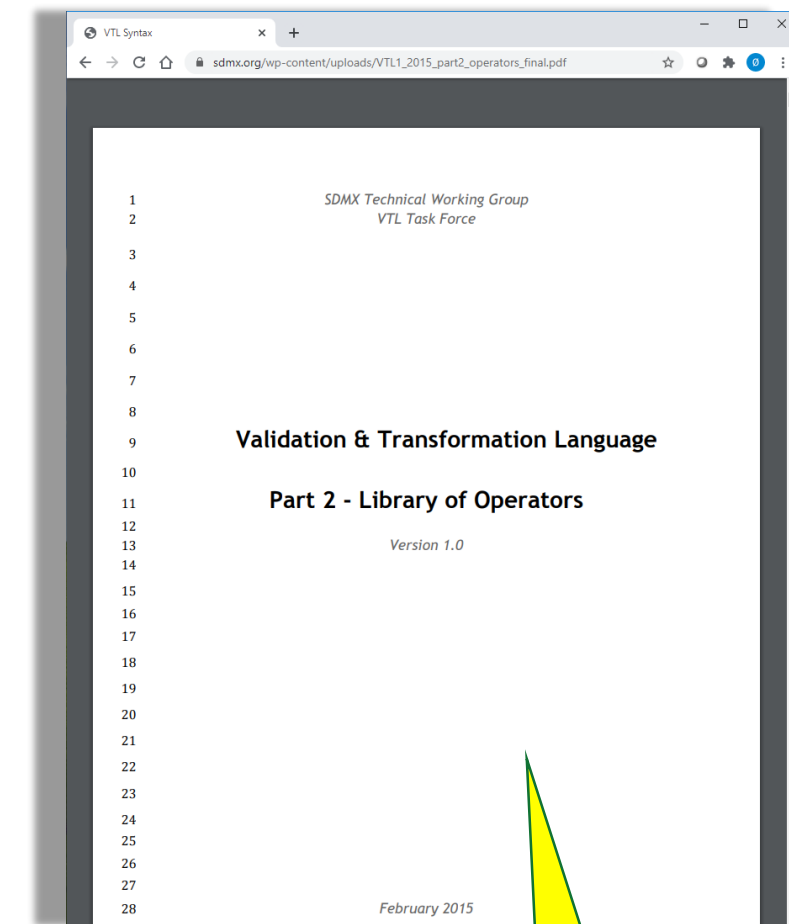
- 6 dimensions

- **Year:** 1 year. We look at one year at a time (year 2016)
- **Region:** 462 municipalities
- **Account category:** 2 categories (not on previous slide)
- **Accounting scope:** 2 scopes
- **Function:** 218 codes defined by a hierarchy
- **Type:** 208 codes defined by a hierarchy

- The challenge

Compute sum values for all 83,795,712 combinations (even if 90% of them are zero) from

- Input file with sum values for 790,527 combinations
 - Only function and type codes from the lowest hierarchy level
 - Missing combinations in input has zero sum
- Hierarchical specifications for function and type



VTL turned out to be too inefficient



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80 Million Challenge



- 6 dimensions
 - **Year:** 1 year. We look at one year at a time (year 2016)
 - **Region:** 462 municipalities
 - **Account category:** 2 categories (not on previous slide)
 - **Accounting scope:** 2 scopes
 - **Function:** 218 codes defined by a hierarchy
 - **Type:** 208 codes defined by a hierarchy

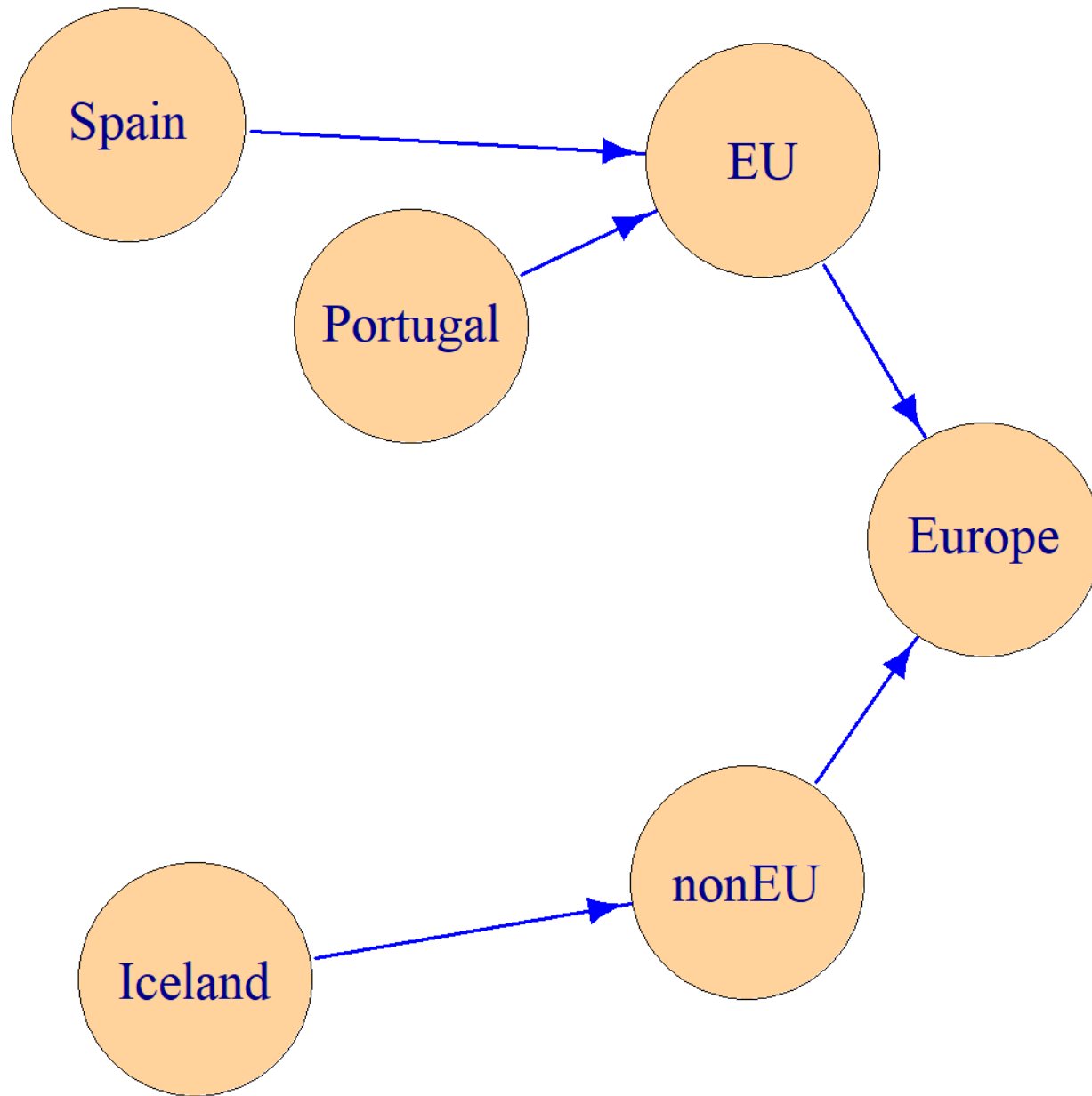
These hierarchies are messy

- The challenge

Compute sum values for all 83,795,712 combinations (even if 90% of them are zero) from

- Input file with sum values for 790,527 combinations
 - Only function and type codes from the lowest hierarchy level
 - Missing combinations in input has zero sum
- Hierarchical specifications for function and type





formulas

Europe = EU + nonEU

EU = Portugal + Spain

nonEU = Iceland

These are more general

- A tree structure not needed
- Sign can be negative
- Free hierarchies

formulas

Europe = EU + nonEU

EU = Portugal + Spain

nonEU = Iceland

sdcTable

levels codes

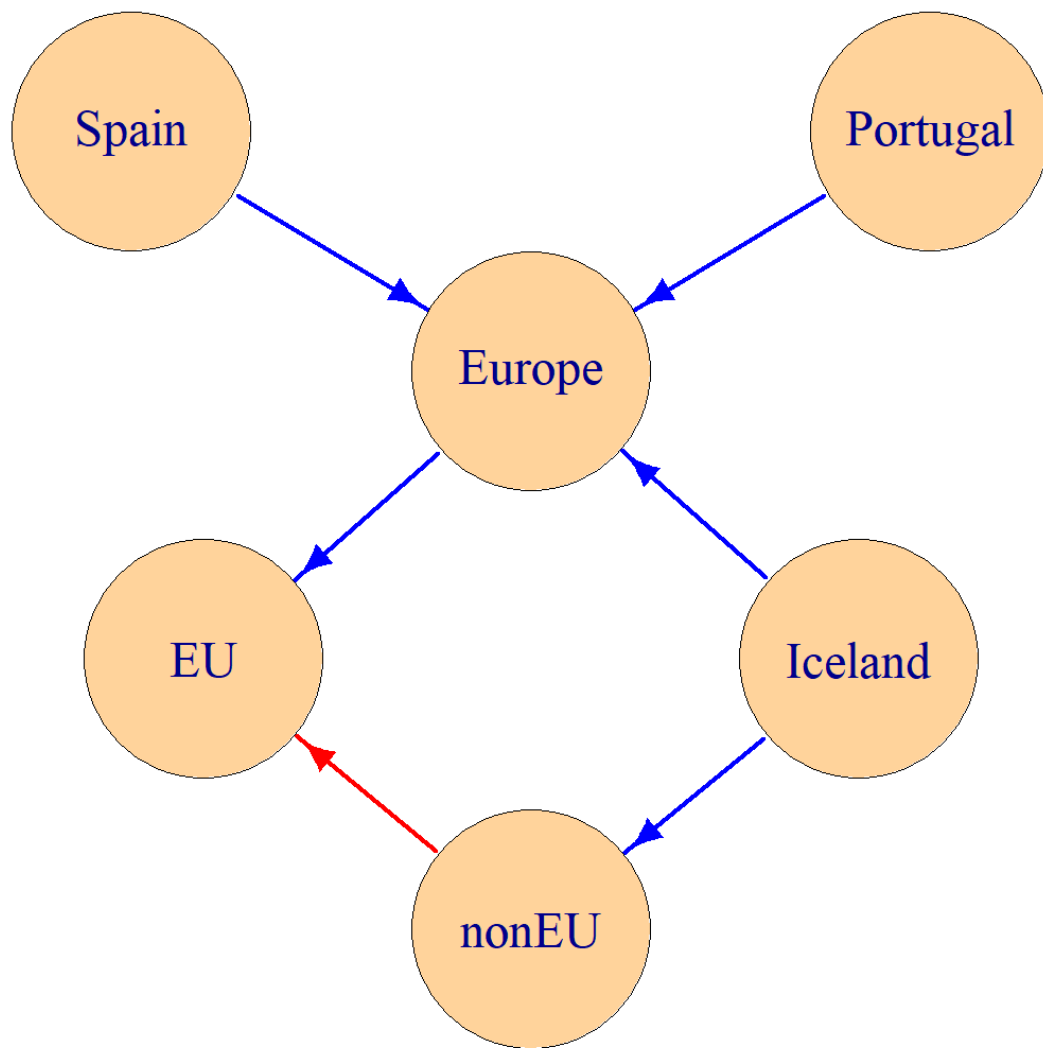
@	Europe
@@	EU
@@@	Portugal
@@@	Spain
@@	nonEU
@@@	Iceland

tauArgus

EU
@Portugal
@Spain
nonEU
@Iceland

mapsFrom	mapsTo	sign	level
EU	Europe	1	2
Portugal	EU	1	1
Spain	EU	1	1
nonEU	Europe	1	2
Iceland	nonEU	1	1





formulas

Europe = Iceland + Portugal + Spain

nonEU = Iceland

EU = Europe - nonEU

mapsFrom	mapsTo	sign	level
Iceland	Europe	1	1
Portugal	Europe	1	1
Spain	Europe	1	1
Iceland	nonEU	1	1
Europe	EU	1	2
nonEU	EU	-1	2



Dummy hierarchy matrix →

- First part of the computation process
 - Input codes as columns. Found in mapsFrom level 1.
 - Matrix created by a matrix multiplication for each level
 - The levels can be found automatically
- Both specifications below give same matrix
- Input codes can also be included as rows
 - Parameter inputInOut = **FALSE**

	Iceland	Portugal	Spain
EU	0	1	1
nonEU	1	0	0
Europe	1	1	1

mapsFrom	mapsTo	sign	level
EU	Europe	1	2
Portugal	EU	1	1
Spain	EU	1	1
nonEU	Europe	1	2
Iceland	nonEU	1	1

mapsFrom	mapsTo	sign	level
Iceland	Europe	1	1
Portugal	Europe	1	1
Spain	Europe	1	1
Iceland	nonEU	1	1
Europe	EU	1	2
nonEU	EU	-1	2

Dummy hierarchy matrix →

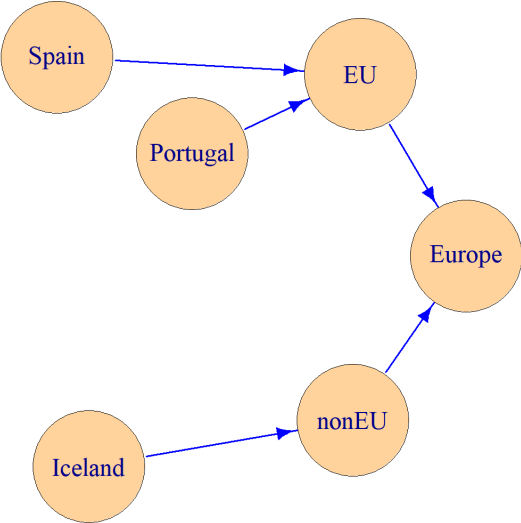
- First part of the computation process
 - Input codes as columns. Found in mapsFrom level 1.
 - Matrix created by a matrix multiplication for each level
 - The levels can be found automatically
- Both specifications below give same matrix
- Input codes can also be included as rows
 - Parameter inputInOut = **TRUE**

mapsFrom	mapsTo	sign	level
EU	Europe	1	2
Portugal	EU	1	1
Spain	EU	1	1
nonEU	Europe	1	2
Iceland	nonEU	1	1

	Iceland	Portugal	Spain
EU	0	1	1
nonEU	1	0	0
Europe	1	1	1
Iceland	1	0	0
Portugal	0	1	0
Spain	0	0	1

mapsFrom	mapsTo	sign	level
Iceland	Europe	1	1
Portugal	Europe	1	1
Spain	Europe	1	1
Iceland	nonEU	1	1
Europe	EU	1	2
nonEU	EU	-1	2

Geographic hierarchy

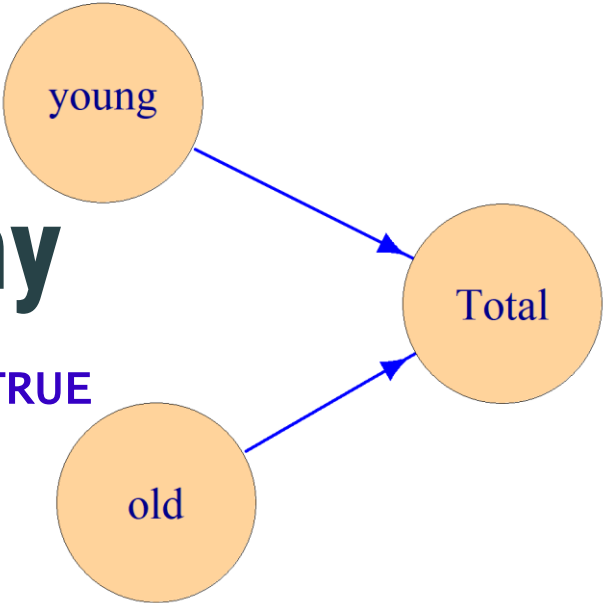


inputInOut = FALSE

mapsFrom	mapsTo	sign	level
EU	Europe	1	2
Portugal	EU	1	1
Spain	EU	1	1
nonEU	Europe	1	2
Iceland	nonEU	1	1

	Iceland	Portugal	Spain
EU	0	1	1
nonEU	1	0	0
Europe	1	1	1

Age hierarchy



inputInOut = TRUE

mapsFrom	mapsTo	sign	level
young	Total	1	1
old	Total	1	1

	young	old
old	0	1
young	1	0
Total	1	1

Input to HierarchyCompute

age	geo	value
young	Spain	66.9
young	Iceland	1.8
young	Portugal	11.6
old	Spain	120.3
old	Iceland	1.5
old	Portugal	20.2

inputInOut = FALSE

mapsFrom	mapsTo	sign	level
EU	Europe	1	2
Portugal	EU	1	1
Spain	EU	1	1
nonEU	Europe	1	2
Iceland	nonEU	1	1

inputInOut = TRUE

mapsFrom	mapsTo	sign	level
young	Total	1	1
old	Total	1	1



> z

age	geo	value
young	Spain	66.9
young	Iceland	1.8
young	Portugal	11.6
old	Spain	120.3
old	Iceland	1.5
old	Portugal	20.2

Computing aggregates by HierarchyCompute

```
> HierarchyCompute(z, list(age =  
ageHier, geo = geoHier), "value",  
inputInOut = c(TRUE, FALSE))
```

age	geo	value
old	EU	140.5
young	EU	78.5
Total	EU	219.0
old	nonEU	1.5
young	nonEU	1.8
Total	nonEU	3.3
old	Europe	142.0
young	Europe	80.3
Total	Europe	222.3



> z

age	geo	value
young	Spain	66.9
young	Iceland	1.8
young	Portugal	11.6
old	Spain	120.3
old	Iceland	1.5
old	Portugal	20.2

Computing aggregates by HierarchyCompute

- Computed by matrix multiplication
- A model matrix (transposed)

```
> HierarchyCompute(z, list(age =  
ageHier, geo = geoHier), "value",  
inputInOut = c(TRUE, FALSE))
```

age	geo	value
old	EU	140.5
young	EU	78.5
Total	EU	219.0
old	nonEU	1.5
young	nonEU	1.8
Total	nonEU	3.3
old	Europe	142.0
young	Europe	80.3
Total	Europe	222.3

=

.	.	.	1	.	1
1	.	1	.	.	.
1	.	1	1	.	1
.	.	.	.	1	.
.	1	.	.	.	.
.	1	.	.	1	.
.	.	.	1	1	1
1	1	1	.	.	.
1	1	1	1	1	1

×

value
66.9
1.8
11.6
120.3
1.5
20.2



Creating the model matrix for the matrix multiplication

- The final model matrix is the result of crossing a separate model matrix for each hierarchy
- Each is a dummy hierarchy matrix extended/replicated to match data
- Crossing by column-wise Kronecker product
 - Function KhatriRao in the Matrix package

age			geo			geo		
young	old	Total	EU	nonEU	Europe	young	old	Total
1	0	0	1	0	0	1	0	0
0	1	0	1	0	0	0	1	0
0	0	1	1	0	0	0	0	1
1	0	0	0	1	0	1	0	0
0	1	0	0	1	0	0	1	0
0	0	1	0	1	0	0	0	1

EU	1	.	1	1	.	1
nonEU	.	1	.	.	1	.
Europe	1	1	1	1	1	1

old	.	.	.	1	1	1
young	1	1	1	.	.	.
Total	1	1	1	1	1	1

	Iceland	Portugal	Spain
EU	0	1	1
nonEU	1	0	0
Europe	1	1	1

	young	old
old	0	1
young	1	0
Total	1	1

HierarchyCompute

Extended example

```
> x
  age    geo year value
young  Spain 2014  66.9
young  Iceland 2014   1.8
young  Portugal 2014  11.6
  old    Spain 2014 120.3
  old    Iceland 2014   1.5
  old    Portugal 2014  20.2
young   Spain 2015  63.4
young   Iceland 2015   1.9
young   Portugal 2015  14.2
  old    Spain 2015 119.6
  old    Iceland 2015   1.6
  old    Portugal 2015  24.3
young   Spain 2016  69.1
young   Iceland 2016   1.9
young   Portugal 2016  12.7
  old    Spain 2016 122.1
  old    Iceland 2016   1.9
  old    Portugal 2016  25.8
```

```
> HierarchyCompute(x, list(age = ageHier, geo = geoHier,
year = "rowFactor"), "value", inputInOut = c(TRUE, FALSE))
```

```
  age    geo year value
  old     EU 2014 140.5
young     EU 2014  78.5
Total     EU 2014 219.0
  old nonEU 2014   1.5
young nonEU 2014   1.8
Total nonEU 2014   3.3
  old Europe 2014 142.0
young Europe 2014  80.3
Total Europe 2014 222.3
  old     EU 2015 143.9
young     EU 2015  77.6
Total     EU 2015 221.5
  old nonEU 2015   1.6
young nonEU 2015   1.9
Total nonEU 2015   3.5
  old Europe 2015 145.5
young Europe 2015  79.5
Total Europe 2015 225.0
  old     EU 2016 147.9
young     EU 2016  81.8
Total     EU 2016 229.7
  old nonEU 2016   1.9
young nonEU 2016   1.9
Total nonEU 2016   3.8
  old Europe 2016 149.8
young Europe 2016  83.7
Total Europe 2016 233.5
```

- year is extra dimension without hierarchy

- Just a factor

- Specified by "rowFactor"

- Here
 - Two hierarchies and one factor
- In general
 - No limits



HierarchyCompute

Extended example

```
> x
  age    geo year value
young  Spain 2014  66.9
young  Iceland 2014   1.8
young  Portugal 2014  11.6
  old   Spain 2014 120.3
  old   Iceland 2014   1.5
  old   Portugal 2014  20.2
young   Spain 2015  63.4
young   Iceland 2015   1.9
young   Portugal 2015  14.2
  old   Spain 2015 119.6
  old   Iceland 2015   1.6
  old   Portugal 2015  24.3
young   Spain 2016  69.1
young   Iceland 2016   1.9
young   Portugal 2016  12.7
  old   Spain 2016 122.1
  old   Iceland 2016   1.9
  old   Portugal 2016  25.8
```

```
> HierarchyCompute(x, list(age = ageHier, geo = geoHier,
year = "colFactor" ), "value", inputInOut = c(TRUE, FALSE))
```

```
  age    geo year value
  old     EU 2014 140.5
young     EU 2014  78.5
Total     EU 2014 219.0
  old nonEU 2014   1.5
young nonEU 2014   1.8
Total nonEU 2014   3.3
  old Europe 2014 142.0
young Europe 2014  80.3
Total Europe 2014 222.3
  old     EU 2015 143.9
young     EU 2015  77.6
Total     EU 2015 221.5
  old nonEU 2015   1.6
young nonEU 2015   1.9
Total nonEU 2015   3.5
  old Europe 2015 145.5
young Europe 2015  79.5
Total Europe 2015 225.0
  old     EU 2016 147.9
young     EU 2016  81.8
Total     EU 2016 229.7
  old nonEU 2016   1.9
young nonEU 2016   1.9
Total nonEU 2016   3.8
  old Europe 2016 149.8
young Europe 2016  83.7
Total Europe 2016 233.5
```

- year is extra dimension without hierarchy

- Just a factor

- Specified by **"colFactor"**

- Same output

- Column order can differ

Change the
specification of
year from
rowFactor to
colFactor



Computations behind the scenes

- Reorganizing to wider format can improve efficiency
- Use `output = "matrixComponents"` to see the matrices

2014	2015	2016
140.5	143.9	147.9
78.5	77.6	81.8
219.0	221.5	229.7
1.5	1.6	1.9
1.8	1.9	1.9
3.3	3.5	3.8
142.0	145.5	149.8
80.3	79.5	83.7
222.3	225.0	233.5

=

.	1	1	.	.	.
.	.	.	.	1	1
.	1	1	.	1	1
1	.	.	.	.	.
.	.	.	1	.	.
1	.	.	1	.	.
1	1	1	.	.	.
.	.	.	1	1	1
1	1	1	1	1	1

×

2014	2015	2016
1.5	1.6	1.9
20.2	24.3	25.8
120.3	119.6	122.1
1.8	1.9	1.9
11.6	14.2	12.7
66.9	63.4	69.1

Holes may occur

- Because of incomplete input
- Holes filled with zeros



80 Million Challenge

- 6 dimensions
 - **Year:** 1 year. We look at one year at a time (year 2016)
 - **Region:** 462 municipalities
 - **Account category:** 2 categories (not on previous slide)
 - **Accounting scope:** 2 scopes
 - **Function:** 218 codes defined by a hierarchy
 - **Type:** 208 codes defined by a hierarchy

region = "colFactor"

These hierarchies are messy

- The challenge

Compute sum values for all 83,795,712 combinations (even if 90% of them are zero) from

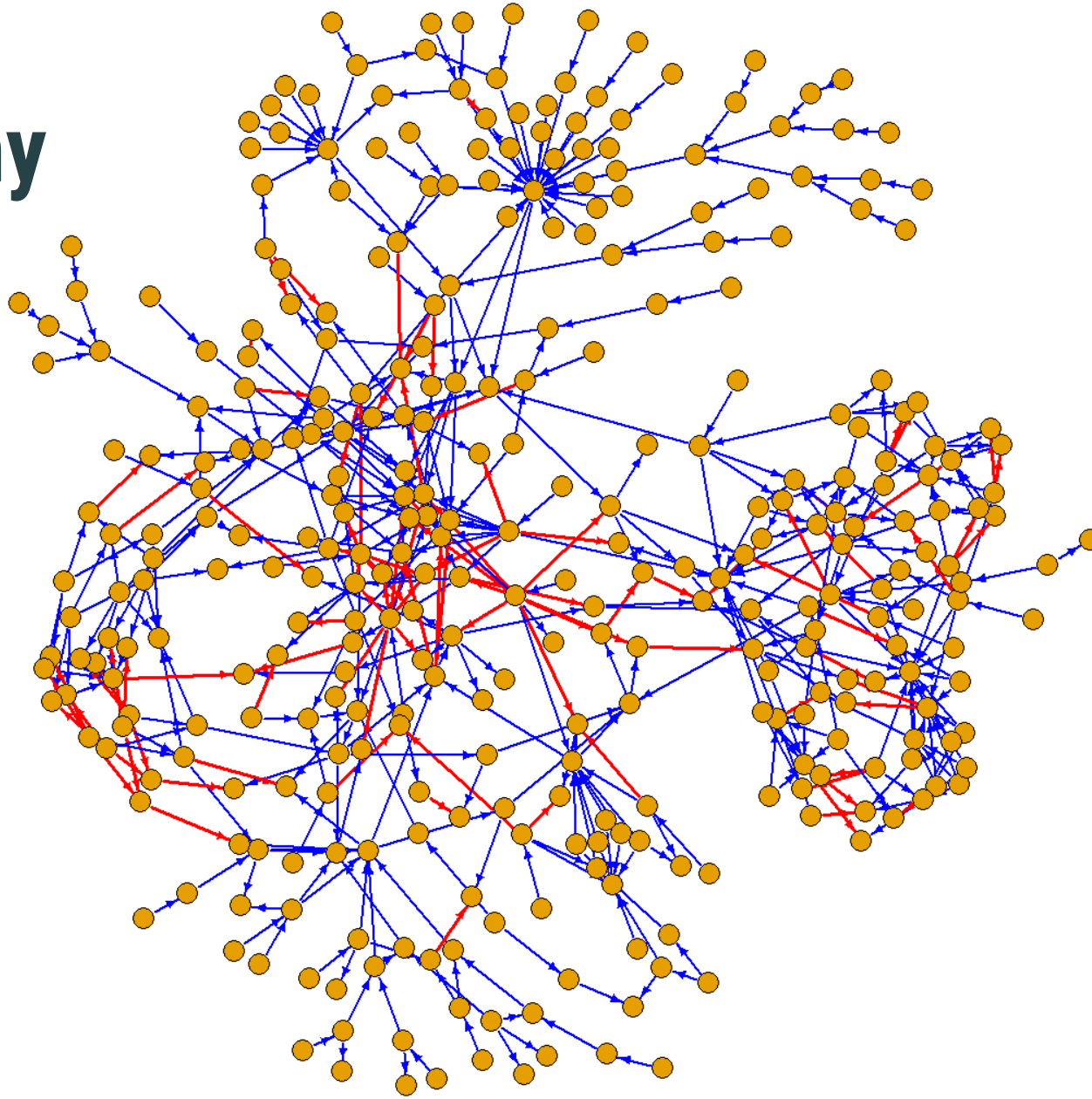
- Input file with sum values for 790,527 combinations
 - Only function and type codes from the lowest hierarchy level
 - Missing combinations in input has zero sum
- Hierarchical specifications for function and type



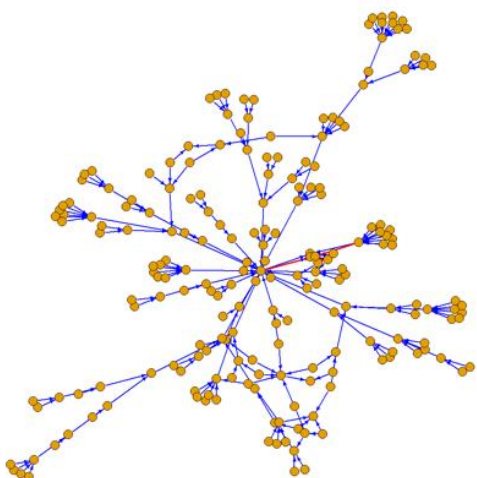
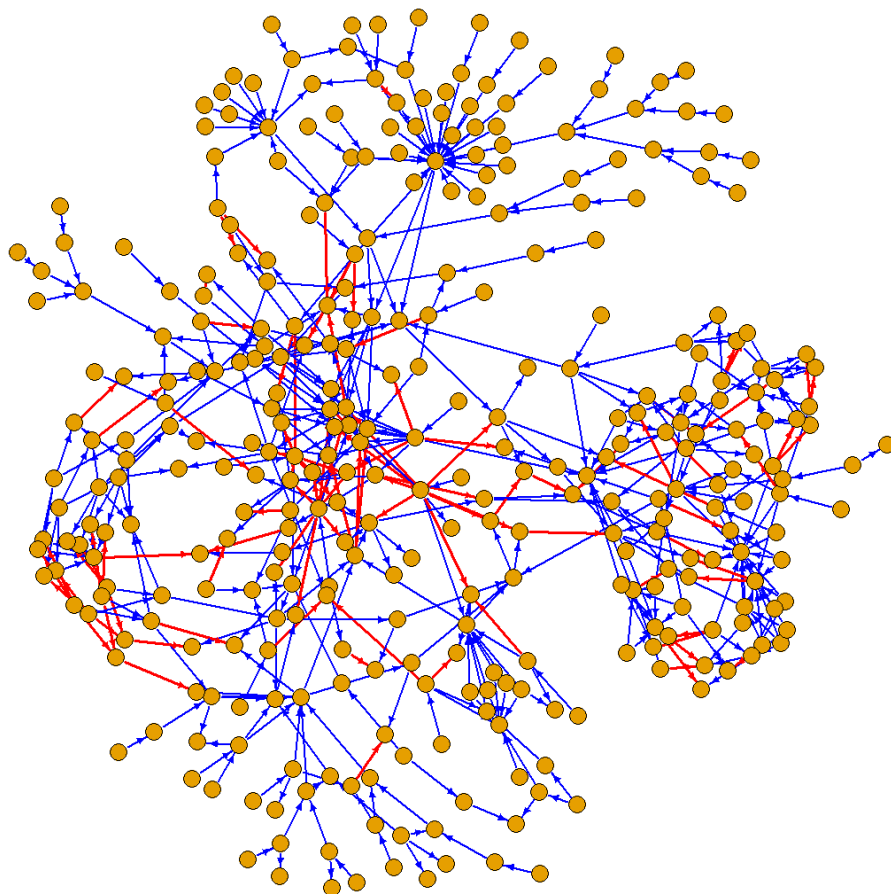
Function hierarchy



Type hierarchy



80 Million Challenge



20 seconds computing time

- On my old laptop
- Of which 10 seconds to organize the matrix results in a data frame



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Extensions and spin-offs - after mission completed

- Computing selected combinations for output
 - Parameters `rowSelect`, `colSelect`, `select`
- Handling of duplicates
 - Parameter `handleDuplicated`
- “Two-way” computation
 - Parameter `colVar` and function `HierarchyCompute2()`
- Model matrix as primary output and combining with formula
 - Functions `Hierarchies2ModelMatrix()` and `HierarchiesAndFormula2ModelMatrix()`
- Allowing hierarchies specified in several ways
 - Function `AutoHierarchies()`
- Making use of model matrices within statistical disclosure control
 - Package `SmallCountRounding`



	EU	nonEU	Europe
old:2014	140.5	1.5	142.0
young:2014	78.5	1.8	80.3
Total:2014	219.0	3.3	222.3
old:2015	143.9	1.6	145.5
young:2015	77.6	1.9	79.5
Total:2015	221.5	3.5	225.0
old:2016	147.9	1.9	149.8
young:2016	81.8	1.9	83.7
Total:2016	229.7	3.8	233.5

$$=$$

1	.	.	.	.	.
.	.	.	1	.	.
1	.	.	1	.	.
.	1	.	.	.	.
.	.	.	.	1	.
.	1	.	.	1	.
.	.	1	.	.	.
.	.	.	.	.	1
.	.	1	.	.	1

$$\times$$

1.5	20.2	120.3
1.6	24.3	119.6
1.9	25.8	122.1
1.8	11.6	66.9
1.9	14.2	63.4
1.9	12.7	69.1

$$\times$$

.	1	1
1	.	1
1	.	1

- Parameter `handleDuplicated`

→ • “Two-way” computation

- Parameter `colVar` and function `HierarchyCompute2()`

• Model matrix as primary output and combining with formula

- Functions `Hierarchies2ModelMatrix()` and `HierarchiesAndFormula2ModelMatrix()`

• Allowing hierarchies specified in several ways

- Function `AutoHierarchies()`

• Making use of model matrices within statistical disclosure control

- Package `SmallCountRounding`



Extensions and spin-offs - after mission completed

```
> Hierarchies2ModelMatrix(x, list(age = ageHier, geo = geoHier, year = ""), inputInOut = c(TRUE, FALSE))  
18 x 27 sparse Matrix of class "dgCMatrix"
```

```
[[ suppressing 27 column names 'Total:Europe:2014', 'Total:Europe:2015', 'Total:Europe:2016' ... ]]
```

```
[1,] 1 . . 1 . . . . . . . . . . . . . . 1 . . 1 . . . . .  
[2,] 1 . . . . . 1 . . . . . . . . . . 1 . . . . 1 . .  
[3,] 1 . . 1 . . . . . . . . . . . . . . 1 . . 1 . . . . .  
[4,] 1 . . 1 . . . . . 1 . . 1 . . . . . . . . . . . . . .  
[5,] 1 . . . . . 1 . . 1 . . . . 1 . . . . . . . . . . .  
[6,] 1 . . 1 . . . . . 1 . . 1 . . . . . . . . . . . . . .  
[7,] . 1 . . 1 . . . . . . . . . . . . . . 1 . . 1 . . . .  
[8,] . 1 . . . . . 1 . . . . . . . . . . 1 . . . . 1 . .  
[9,] . 1 . . 1 . . . . . . . . . . . . . . 1 . . 1 . . . .  
[10,] . 1 . . 1 . . . . . 1 . . 1 . . . . . . . . . . . . .  
[11,] . 1 . . . . . 1 . . 1 . . . . 1 . . . . . . . . . . .  
[12,] . 1 . . 1 . . . . . 1 . . 1 . . . . . . . . . . . . .  
[13,] . . 1 . . 1 . . . . . . . . . . . . . . 1 . . 1 . . .  
[14,] . . 1 . . . . . 1 . . . . . . . . . . . 1 . . . . 1 .  
[15,] . . 1 . . 1 . . . . . . . . . . . . . . 1 . . 1 . . .  
[16,] . . 1 . . 1 . . . . . 1 . . 1 . . . . . . . . . . . .  
[17,] . . 1 . . . . 1 . . 1 . . . . . 1 . . . . . . . . . .  
[18,] . . 1 . . 1 . . . . . 1 . . 1 . . . . . . . . . . . .
```



sdCtable	
<u>levels</u>	<u>codes</u>
@	Europe
@@	EU
@@@	Portugal
@@@	Spain
@@	nonEU
@@@	Iceland

tauArgus
EU
@Portugal
@Spain
nonEU
@Iceland

mapsFrom	mapsTo	sign	level
EU	Europe	1	2
Portugal	EU	1	1
Spain	EU	1	1
nonEU	Europe	1	2
Iceland	nonEU	1	1

formulas

Europe = EU + nonEU

EU = Portugal + Spain

nonEU = Iceland

→ • Allowing hierarchies specified in several ways

- Function `AutoHierarchies()`
- Making use of model matrices within statistical disclosure
- Package `SmallCountRounding`



```
e6 <- SmallCountData("e6") # As Table 4
eDimList <- SmallCountData("eDimList")
eDimList
```

```
$geo
  levels codes
1      @   Total
2     @@    EU
3    @@@ Portugal
4    @@@   Spain
5     @@  nonEU
6    @@@  Iceland
```

```
$year
  levels codes
1      @ Total
2     @@ 2018
3     @@ 2019
```



Table 5: Hierarchy, geo

levels	codes
@	Total
@@	EU
@@@	Portugal
@@@	Spain
@@	nonEU
@@@	Iceland

TauArgus coding. More general coding is also possible. See ?AutoHierarchies

case were all possible combinations (including the inner cells) are to be example we use 5 as a rounding base. As can be seen below, this output can al ways. The inner cells are colored according to the rounding.

Table 6: Ouput data (publish)

geo	year	original	rounded	difference
Total	Total	24	23	-1
Total	2018	12	12	0
Total	2019	12	11	-1
EU	Total	21	23	2
EU	2018	10	12	2
EU	2019	11	11	0
nonEU	Total	3	0	-3
nonEU	2018	2	0	-2
nonEU	2019	1	0	-1
Iceland	Total	3	0	-3
Iceland	2018	2	0	-2
Iceland	2019	1	0	-1
Portugal	Total	8	10	2
Portugal	2018	3	5	2
Portugal	2019	5	5	0
Spain	Total	13	13	0
Spain	2018	7	7	0
Spain	2019	6	6	0

As seen above, a hierarchy is specified for both variables. `eDimList$geo` is given in Table 5 and `eDimList$year` is a plain hierarchy with total code.

Four ways to produce Table 6

The four lines below produce the same results with element `publish` as in Table 6. Ordering of rows can be different.

```
PLSrounding(e6, "freq", 5) # a)
PLSrounding(e6, "freq", 5, formula = ~eu * year + geo * year) # b)
PLSrounding(e6[, -2], "freq", 5, hierarchies = eDimList) # c)
PLSrounding(e6[, -2], "freq", 5, hierarchies = eDimList, formula = ~geo * year) # d)
```

- In a) and c) the function uses hierarchies for the calculations. In a) the hierarchies are found automatically from the input data.
- In b) the cross-classifications are found from the formula. In addition, hierarchical relations in the input data are analysed so that `geo` and `eu` are combined into the same output column.
- In d), how to cross the hierarchies are defined by a formula.

Conclusion

- Sparse matrix tools in the R package **SSBtools**
 - Depends on R base package Matrix
- Main function **HierarchyCompute()**
- Other functions
 - **Hierarchies2ModelMatrix()**
 - **HierarchiesAndFormula2ModelMatrix()**
 - **AutoHierarchies()**
 - **FindHierarchies()**
 - .

